State Space Adaptive Control for a Lumped Mass Rotor Excited by Nonconservative Cross-Coupling Forces

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Abstract

An adaptive pole-assignment controller is presented in conjunction with a state space model identification for an active magnetic bearing system, consisting of a single mass suspended in an active magnetic bearing excited by nonconservative cross-coupling forces. Simulation results show the success of this algorithm for parameter changes in closed loop operation with system noise but without set point changes.

1 Introduction

In conventional rotor-bearing systems with journal bearings and seals, nonconservative cross-coupling forces may lead to instability. A possible way to cope with this problem is to reduce the influence of the cross-coupling mechanism by applying higher damping to the system, which means dissipation of energy.

If a rotor is levitated by magnetic bearings, one can make use of the controllability of the entire system by active magnetic bearings. Thus it is possible to control nonconservative cross-coupling forces by means of on-line identification and adaptive control techniques. It has turned out, that a state space model identification is appropriate to estimate both system parameters and all states. Additionally, a state space model is a good basis for the calculation of a controller for an unstable system. In this case pole-assignment is the best way for an adaptive controller concerning the future implementation in a real-time application.

Finally, using this algorithm, nonconservative forces can be compensated by control forces without loss of stability and without applying more damping to the rotor-bearing system.

2 State space rotor model

The continuous time state space model of a single mass suspended by an active magnetic bearing (see Figure 1) is given by

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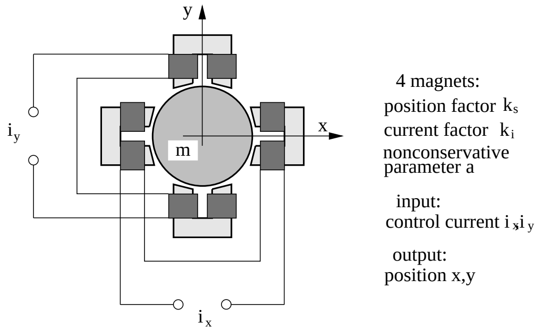

Figure 1: Single mass suspended by active magnetic bearings

$$\dot{\mathbf{z}} = \mathcal{A} \cdot \mathbf{z} + \mathcal{B} \cdot \mathbf{u} \quad (1)$$

$$\mathbf{y} = \mathcal{C} \cdot \mathbf{z} \quad (2)$$

with the system matrix

$$\mathcal{A} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m} & \frac{a}{m} & 0 & 0 \\ -\frac{a}{m} & \frac{k_s}{m} & 0 & 0 \end{pmatrix},$$

the control matrix

$$\mathcal{B} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \frac{k_i}{m} & 0 \\ 0 & \frac{k_i}{m} \end{pmatrix},$$

the measurement matrix

$$\mathcal{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

and the state vector $\mathbf{z} = [x, y, \dot{x}, \dot{y}]^T$, respectively. The lumped mass of the rotor is m. An active magnetic bearing is modelled by the position factor k_s and the current factor k_i. The variable a represents the nonconservative cross-coupling parameter. A transformation to a discrete time system with sampling time T_s yields a model with no particular structure:

$$\mathbf{A} = e^{\mathcal{A} \cdot T_s} \quad (3)$$

$$\mathbf{B} = (e^{\mathcal{A} \cdot T_s} - \mathbf{I}) \cdot \mathcal{A}^{-1} \cdot \mathcal{B} \quad (4)$$

The entire model with system noise $\boldsymbol{\xi}(k)$ and measurement noise $\boldsymbol{\eta}(k)$ can then be written in the form:

$$\mathbf{x}(k+1) = \mathbf{A}\,\mathbf{x}(k) + \mathbf{B}\,\mathbf{u}(k) + \boldsymbol{\xi}(k) \quad (5)$$

$$\mathbf{y}(k) = \mathbf{C}\,\mathbf{x}(k) + \boldsymbol{\eta}(k) \quad (6)$$

Since the discrete state space model should have a minimum number of parameters to be identified, it is transformed into a canonical form. With respect to the calculation of an adaptive controller, a controller canonical form is chosen. The structural indices are for reasons of symmetry $n_1 = 2$ and $n_2 = 2$. The transformation

$$\mathbf{x}_C = \mathbf{\Gamma}_C \, \mathbf{x} \quad \text{and} \quad \mathbf{x} = \mathbf{\Gamma}_C^{-1} \, \mathbf{x}_C \tag{7}$$

can be performed using the transformation matrix

$$\mathbf{\Gamma}_C = \left(\boldsymbol{\gamma}_1, \mathbf{A}^T \boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \mathbf{A}^T \boldsymbol{\gamma}_2\right)^T \tag{8}$$

with its vectors

$$\boldsymbol{\gamma}_i = \left(\mathbf{T}_C^T\right)^{-1} \mathbf{B}_C(:, i), \tag{9}$$

the controller matrix

$$\mathbf{T}_C = \left[\mathbf{B}(:,1), \mathbf{A}\,\mathbf{B}(:,1), \mathbf{B}(:,2), \mathbf{A}\,\mathbf{B}(:,2)\right], \tag{10}$$

and the desired control matrix

$$\mathbf{B}_C = \mathbf{\Gamma}_C \, \mathbf{B} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

with the expression in parenthesis $(:, i)$ denoting the i^{th} column with $i = 1, 2$. This will result in the new system matrix,

$$\mathbf{A}_C = \mathbf{\Gamma}_C \, \mathbf{A} \, \mathbf{\Gamma}_C^{-1} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -a_2^{(11)} & -a_1^{(11)} & -a_2^{(12)} & -a_1^{(12)} \\ 0 & 0 & 0 & 1 \\ -a_2^{(21)} & -a_1^{(21)} & -a_2^{(22)} & -a_1^{(22)} \end{pmatrix},$$

and a new measurement matrix,

$$\mathbf{C}_R = \mathbf{C} \, \mathbf{\Gamma}_C^{-1} = \begin{pmatrix} c_2^{(11)} & c_1^{(11)} & c_2^{(12)} & c_1^{(12)} \\ c_2^{(21)} & c_1^{(21)} & c_2^{(22)} & c_1^{(22)} \end{pmatrix}.$$

3 Identification algorithm

The identification algorithm is based on the innovations model ([3]):

$$\hat{\mathbf{x}}(k+1, \mathbf{p}) = \mathbf{A}(\mathbf{p}) \, \hat{\mathbf{x}}(k, \mathbf{p}) + \mathbf{B}(\mathbf{p}) \, \mathbf{u}(k) + \mathbf{K}(\mathbf{p}) \left[\mathbf{y} - \mathbf{C}(\mathbf{p}) \, \hat{\mathbf{x}}(k, \mathbf{p})\right], \tag{11}$$

$$\hat{\mathbf{y}}(k, \mathbf{p}) = \mathbf{C}(\mathbf{p}) \, \hat{\mathbf{x}}(k, \mathbf{p}) \tag{12}$$

using a Kalman filter matrix

$$\mathbf{K} = \begin{pmatrix} k_2^{(11)} & k_2^{(12)} \\ k_1^{(11)} & k_1^{(12)} \\ k_2^{(21)} & k_2^{(22)} \\ k_1^{(21)} & k_1^{(22)} \end{pmatrix} \tag{13}$$

All parameters of this innovations model are gathered within the parameter vector $\mathbf{p}$. The identification algorithm ([1, 2, 3]) can be summarised as:

Prediction error

$$\boldsymbol{\varepsilon} = \mathbf{y} - \hat{\mathbf{y}}(k, \mathbf{p}). \tag{14}$$

Adaptation of error matrix

$$\mathbf{L}(k) = \mathbf{P}(k-1)\,\mathbf{\Psi}(k)\,\left[\mathbf{I} + \mathbf{\Psi}^T(k)\,\mathbf{P}(k-1)\,\mathbf{\Psi}(k)\right]^{-1}. \tag{15}$$

Parameter update

$$\hat{\mathbf{p}}(k) = \hat{\mathbf{p}}(k-1) + \mathbf{L}(k)\,\boldsymbol{\varepsilon}. \tag{16}$$

Update of covariance matrix

$$\mathbf{P}(k) = \frac{1}{\rho(k)}\,\left[\mathbf{P}(k-1) - \mathbf{L}(k)\,\mathbf{\Psi}^T(k)\mathbf{P}(k-1)\right]. \tag{17}$$

Based on the new model, a prediction for the next time step can be made in the form

$$\hat{\mathbf{x}}(k+1) = \mathbf{A}(\mathbf{p}(k))\,\hat{\mathbf{x}}(k) + \mathbf{B}(\mathbf{p}(k))\,\mathbf{u}(k) + \mathbf{K}(\mathbf{p}(k))\,\boldsymbol{\varepsilon}, \tag{18}$$

$$\hat{\mathbf{y}}(k+1) = \mathbf{C}(\mathbf{p}(\mathbf{k}))\,\hat{\mathbf{x}}(k+1). \tag{19}$$

The gradient matrix $\mathbf{\Psi}$ is derived from the partial derivative of the estimation error as

$$\mathbf{\Psi}(k) = -\left(\frac{\partial}{\partial \mathbf{p}}\,\boldsymbol{\varepsilon}(k)\right)^T = \left(\frac{\partial \hat{\mathbf{y}}(k, \mathbf{p})}{\partial \mathbf{p}}\right)^T, \tag{20}$$

and is calculated recursively by

$$\begin{aligned} \mathbf{W}(k+1, \hat{\mathbf{p}}) &= [\mathbf{A}(\hat{\mathbf{p}}(k)) - \mathbf{K}(\hat{\mathbf{p}}(k))\,\mathbf{C}(\hat{\mathbf{p}}(k))]\;\mathbf{W}(k, \hat{\mathbf{p}}) \\ &\quad + \mathbf{M}_k - \mathbf{K}(\hat{\mathbf{p}}(k))\,\mathbf{V}_k, \end{aligned} \tag{21}$$

$$\mathbf{\Psi}(k+1) = \mathbf{W}^T(k+1, \hat{\mathbf{p}})\,\mathbf{C}(\hat{\mathbf{p}}(k)) + \mathbf{V}_k^T. \tag{22}$$

The matrices $\mathbf{M}_k$ and $\mathbf{V}_k$ are model specific and result from the partial derivative of the innovations model as

$$\mathbf{M}_k = \frac{\partial}{\partial \mathbf{p}}\left[\mathbf{A}(\mathbf{p})\,\hat{\mathbf{x}}(k) + \mathbf{B}(\mathbf{p})\,\mathbf{u}(k) + \mathbf{K}(\mathbf{p})\,\boldsymbol{\varepsilon}\right], \tag{23}$$

$$\mathbf{V}_k = \frac{\partial}{\partial \mathbf{p}}\left[\mathbf{C}(\mathbf{p})\,\hat{\mathbf{x}}(k)\right]. \tag{24}$$

Special attention has to be paid to the forgetting factor $\rho(k)$ in equation 17. If this factor is too small, the covariance matrix will increase too fast, if it is too large (close to 1), the algorithm will react too slowly to parameter changes. Therefore, the forgetting factor needs to be controlled separately by a statistical value, which gives the change of the variance of the estimation error

$$\delta(k) = \frac{\hat{\sigma}_\varepsilon^2(N_1, k) - \hat{\sigma}_\varepsilon^2(N_2, k)}{\hat{\sigma}_\varepsilon^2(N_2, k)}, \tag{25}$$

with $N_1 \leq N_2$ and $\hat{\sigma}_\varepsilon^2(N, k)$ as estimate for the error variance with a variable memory N. The idea behind this algorithm is, that under the assumption of a stationary process, the variance of the estimation error in a steady state should be stationary as well. The increasing variance of the estimation error indicates a change in parameters of the plant under investigation. If $\delta(k)$ triggers a certain threshold, e.g. 20% of the mean variance, then the forgetting factor $\rho(k)$ is reset to ρ_0 and follows the law such that

$$\rho(k) = k_\rho\, \rho(k-1) + (1 - k_\rho)\, \rho_\infty \tag{26}$$

with ρ_∞ the final value and k_ρ determining the rate of change.

4 Adaptive control by pole placement

The control law is defined as

$$\mathbf{u} = -\mathbf{K}_x\, \mathbf{x} \tag{27}$$

with

$$\mathbf{K}_x = \begin{pmatrix} k_2^{(11)} & k_1^{(11)} & k_2^{(12)} & k_1^{(12)} \\ k_2^{(21)} & k_1^{(21)} & k_2^{(22)} & k_1^{(22)} \end{pmatrix}$$

The poles of the closed loop system $\mathbf{A}_{cl} = \mathbf{A}_C - \mathbf{B}\mathbf{K}_x$ result from

$$P(z) = \begin{vmatrix} z & -1 & 0 & 0 \\ k_2^{(11)} - a_2^{(11)} & z + k_1^{(11)} - a_1^{(11)} & k_2^{(12)} - a_2^{(12)} & k_1^{(12)} - a_1^{(12)} \\ 0 & 0 & z & -1 \\ k_2^{(21)} - a_2^{(21)} & k_1^{(21)} - a_1^{(21)} & k_2^{(22)} - a_2^{(22)} & z + k_1^{(22)} - a_1^{(22)} \end{vmatrix}.$$

Here, the advantage of a model description in controller canonical form becomes obvious. If the cross coupling terms are zero by selecting the cross coupling parameters of the control matrix properly, the characteristic polynomial can be reduced to

$$\begin{aligned} P(z) &= \left(z^2 + (k_1^{(11)} - a_1^{(11)})z + k_2^{(11)} - a_2^{(11)}\right) \\ &\quad \left(z^2 + (k_1^{(22)} - a_1^{(22)})z + (k_2^{(22)} - a_2^{(22)})\right). \end{aligned}$$

If this polynomial is compared with the desired one as

$$P_s(z) = \left(z^2 + p_1 z + p_2\right)^2,$$

then all coefficients of the control matrix can be easily calculated in the form

$$k_m^{(ij)} = a_m^{(ij)}, \tag{28}$$

$$k_m^{(ii)} = a_m^{(ii)} + p_m. \tag{29}$$

with $m = 1, 2$ and $i, j = 1, 2$.

5 Simulation results

The parameters of the system depicted in Figure 1 are given by the following

Parameter	Value	Unit
Rotor mass	28.77	kg
Position factor	$2.39 \cdot 10^6$	N/m
Current factor	299.57	N/A
Nonconservative factor	a	N/m
Sampling time	10^{-4}	s

Further investigations have shown, that the nonconservative stiffness parameter a exclusively affects one parameter of the system matrix, namely $a_1^{(12)}$, which in the following is referred to as p. Additionally, one can make use of the symmetry in this system, and reduce the number of parameters of the Kalman matrix to 4. Thus, only 5 entries remain in the parameter vector $\mathbf{p}$.

The resulting state space model is then

$$
\begin{aligned}
\mathbf{x}(k+1) &= \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 2.01 & 0 & p \\ 0 & 0 & 0 & 1 \\ 0 & -p & -1 & 2.01 \end{pmatrix} \mathbf{x}(k) + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{u}(k) + \boldsymbol{\xi}(k) \\
\mathbf{y}(k) &= \begin{pmatrix} 0.52 \cdot 10^{-7} & 0.52 \cdot 10^{-7} & 0 & 0 \\ 0 & 0 & 0.52 \cdot 10^{-7} & 0.52 \cdot 10^{-7} \end{pmatrix} \mathbf{x}(k) + \boldsymbol{\eta}(k).
\end{aligned}
$$

After the model has been defined with all parameters to be identified, the matrices $\mathbf{M}_k$ and $\mathbf{V}_k$ have to be calculated once according to equation 23 and 24. With these matrices the identification algorithm structure can be established (see equation 14 to 22). Since the system is open loop unstable, a controller and an observer have to be designed in advance. This has been done according to the nominal system without cross-coupling influence. The poles of the closed loop system have been placed at $z_{1,2,3,4} = 0.97$, or in the continuous complex plane to $s_{1,2,3,4} = -288\text{s}^{-1}$, which would meet the eigenvalue for a mass-spring-system with a negative stiffness factor of the magnetic bearing. The resulting control law is

$$
\mathbf{u} = \begin{pmatrix} -0.0591 & 0.0608 & 0 & 0 \\ 0 & 0 & -0.0591 & 0.0608 \end{pmatrix} \mathbf{x}.
$$

Similar to the controller, the observer poles have been chosen to $z_{1,2,3,4} = 0.7$, to ensure, that the observer is fast enough to follow the system behaviour. The Kalman matrix in equation 13 is assigned to

$$
\mathbf{K} = \begin{pmatrix} 5.3322 \cdot 10^6 & 6.2067 \cdot 10^6 & 0 & 0 \\ 0 & 0 & 5.3322 \cdot 10^6 & 6.2067 \cdot 10^6 \end{pmatrix}^T.
$$

The initial parameter vector is set to the a priori values

$$
\mathbf{p} = \begin{pmatrix} 0 & 5.3322 \cdot 10^6 & 6.2067 \cdot 10^6 & 0 & 0 \end{pmatrix}^T,
$$

with the initial covariance matrix $\mathbf{P}$ as identity matrix. All other recursively calculated matrices have been set to zero. White noise with a maximum deflection value of $1\mu m$ for $\boldsymbol{\eta}$ was added to the measurement signal to ensure excitation. With the initial conditions $\mathbf{x} = \mathbf{0}$ and all precalculated values for the controller and observer the simulation was started. The resulting displacement of the rotor mass can be seen in Figures 2, 3, 4.

After 0.01s the parameter a changes from 0 to $1 \cdot 10^7$N/m, which corresponds to a value for $p = 3.4766 \cdot 10^{-3}$. This is assumed to be the worst case in a turbomachinery application, e.g. a sudden pressure loss or leakage in sealings will lead to a parameter change in a linear model. Actually, the mechanism of the cross-coupling forces develops slowly, but an abrupt change in the nonconservative stiffness parameter should cover most cases. Immediately after the parameter change the system is unstable in terms of its structure, but the displacements will increase slowly. When a certain signal to noise ratio is achieved, the parameter change is recognised by the algorithm. This means, that the input and output signals have to contain a minimum of information to make the algorithm converge to the new system parameters.

Parallel to the identification, the controller parameters are calculated according to equation 28 and 29. It can be observed, that the feedback gain matrix of the state space controller becomes non-symmetric to the same extent as the parameter of the nonconservative stiffness does. This effect can easily be seen in equation 28. These non-symmetric entries in the controller matrix will cause a force, which directly compensates the nonconservative forces of the system. Due to the adaptation of the controller the rotor bearing system can recover from a parameter change. Simulations have shown, that this positive effect is true for a certain amount of the non conservative forces. If the parameter change is too large (10^8N/m), the control current becomes to large as well and runs into saturation. Then, instability can't be prevented.

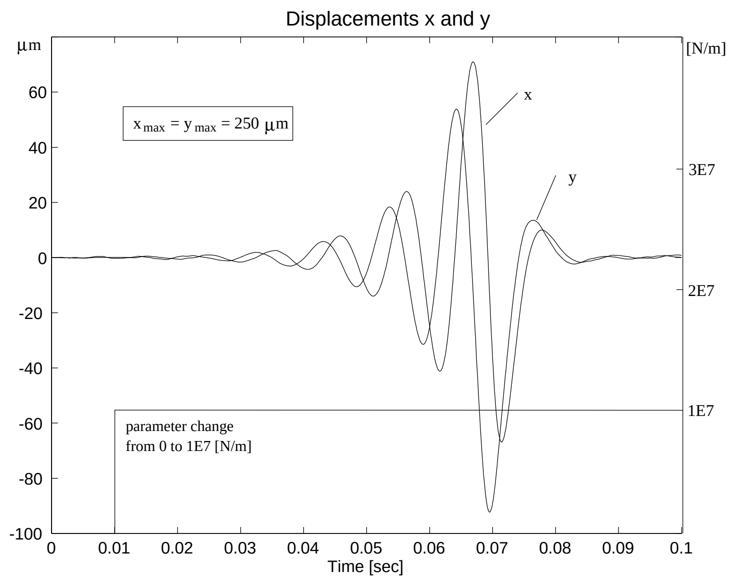

Figure 2: Time history of vertical (x) and horizontal (y) rotor displacements after an abrupt cross-coupling excitation of $a = 10^7$N/m.

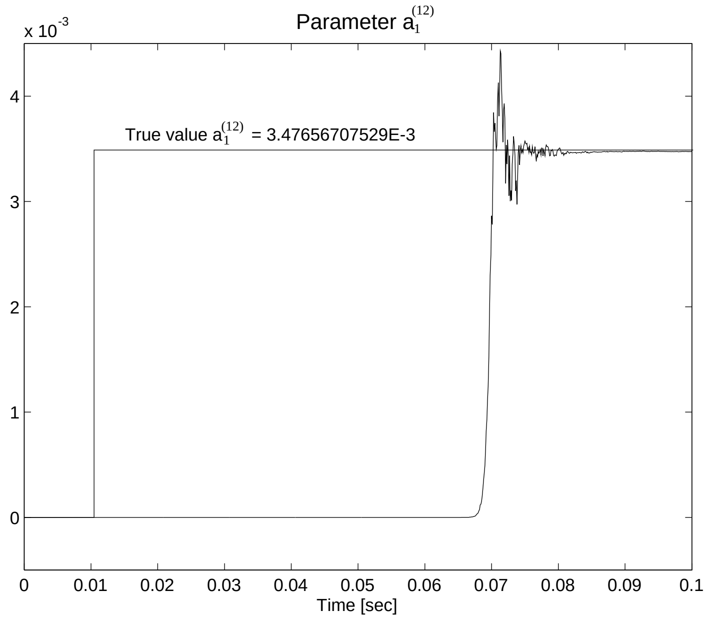

Figure 3: Time history of $\mathbf{p}(1)$. The parameters of the Kalman matrix $\mathbf{K}$ did not change significantly.

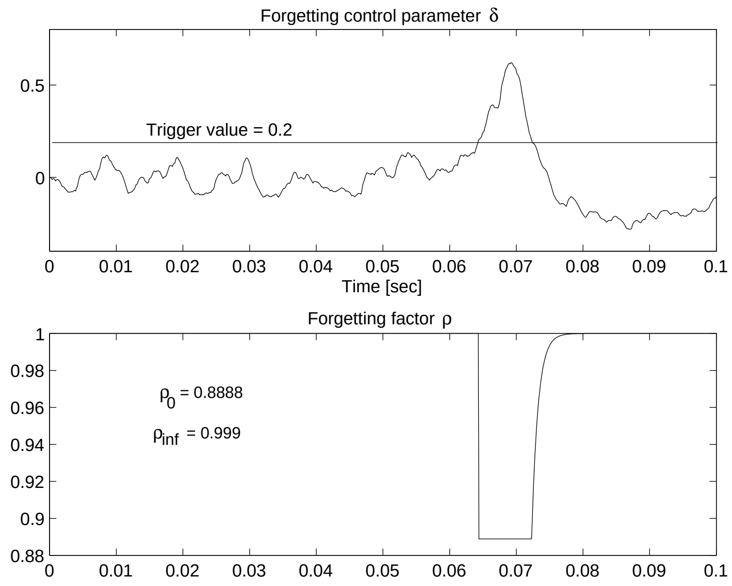

Figure 4: Time history of the forgetting control value δ and the forgetting factor ρ. Approximately after 0.054s the parameter change of the cross-coupling excitation is recognised.

6 Conclusion

It has been shown, that nonconservative cross-coupling forces can be controlled by means of on-line identification and adaptive control techniques. The system under investigation was a lumped mass model of a rigid rotor suspended by an active magnetic bearing controlled by a state space controller. Within a simulation a nonconservative cross coupling force was applied to the system in conjunction with the proposed on-line identification and adaptive control. It has turned out, that a state space model identification is appropriate to estimate both the system parameters and the state vector. In this case, the nonconservative cross-coupling forces were compensated by additional forces generated by a non-symmetric gain in the state space controller matrix. This matrix is calculated by pole assignment from the identified state space model.

Future objectives of investigation will be the extension of this concept for a full identification and central control for a rigid rotor suspended in two magnetic bearings.

7 Acknowledgements

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